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Recent Progress on Nonlinear Integrability

Theoretical work, simulations and ~~experimental results~~

Alexander Valishev

ICFA Mini-Workshop on Nonlinear Dynamics and Collective Effects in Particle Beam Dynamics – NOCE2017. Arcidosso, Italy

21 September 2017

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Modern Accelerator Lattices

- The **backbone of all present accelerators – linear focusing lattice**: particles have nearly identical betatron tunes by design. Such machines are built using dipoles and quadrupoles.
  - **All nonlinearities** (both natural and specially introduced) **are perturbations**

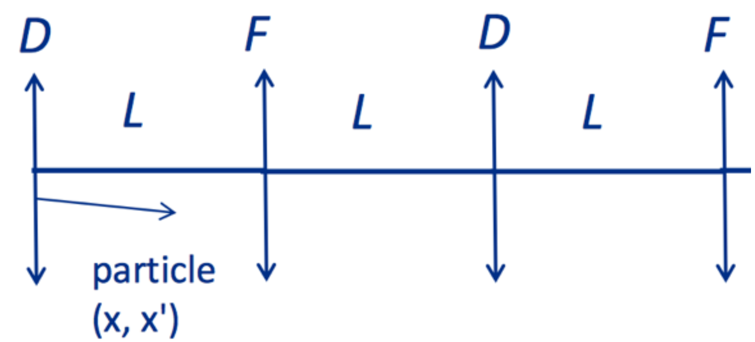
Strong Focusing – Standard Approach Since 1952

Christofilos (1949); Courant, Livingston and Snyder (1952)

$$H = c\left[m^2c^2 + \left(\mathbf{p} - \frac{e}{c}\mathbf{A}\right)^2\right]^{\frac{1}{2}}$$

$$H' \approx \frac{p_x^2 + p_y^2}{2} + \frac{K_x(s)x^2}{2} + \frac{K_y(s)y^2}{2}$$

$$\begin{cases} x'' + K_x(s)x = 0 \\ y'' + K_y(s)y = 0 \end{cases}$$



$$K_{x,y}(s + C) = K_{x,y}(s)$$ -- piecewise constant alternating-sign functions

Stability of Linear Lattices in Light Sources

Low beam emittance requires Strong Focusing

- Strong Focusing leads to strong chromatic aberrations
- Chromaticity correction demands strong sextupole magnets
- Strong sextupoles introduce significant nonlinear perturbation – integrability is ruined
- Dynamical aperture limitation leads to reduced lifetime, issues with injection, etc.

There are ways to (partially) mitigate these issues

- High periodicity
- Proper arrangement of sextupole phase advances

Stability of Linear Lattices in High Intensity Machines

COLLIDING BEAMS: PRESENT STATUS; AND THE SLAC PROJECT*

B. Richter

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

The discovery in the early '60's at the Princeton-Stanford ring of what was thought to be the resistive wall instability brought the realization that circular accelerators are fundamentally unstable devices because of the interaction of the beam with its environment. Stability is achieved only through Landau damping and/or some external damping system.

- **1965 Priceton-Stanford CBX: First mention of an 8-pole magnet**
  - Observed vertical resistive wall instability
  - With octupoles, increased beam current from ~5 to 500 mA
- **CERN PS: In 1959 had 10 octupoles; not used until 1968**
  - At 10^{12} protons/pulse observed (1^{st} time) head-tail instability. Octupoles helped.
  - Once understood, chromaticity jump at transition was developed using sextupoles.
  - More instabilities were discovered; helped by octupoles, fb
- **LHC has 336 octupoles that are used at close to full current**

Focusing: Linear vs. Nonlinear

- Accelerators are linear systems by design (frequency is independent of amplitude).
- In accelerators, nonlinearities are either intentionally introduced (chromaticity correction sextupoles, Landau damping octupoles) or unavoidable (space charge, beam-beam, magnet imperfections).
- All nonlinearities (in present rings) lead to resonances and dynamic aperture limits.

- **Are there "magic" nonlinearities with zero resonance strength?**
- **The answer is – *yes* (we call them "*integrable*")**

Do Accelerators Need to be Linear?

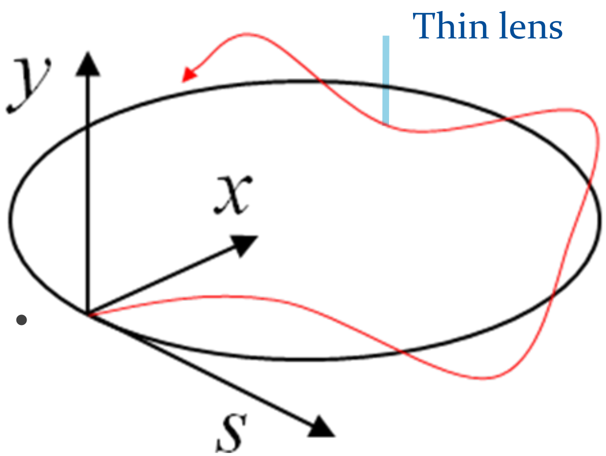
Search for solutions that are strongly nonlinear yet stable

- Orlov (1963)
- McMillan (1967) – 1D solution
- ✓ Perevedentsev, Danilov (1990) – generalization of McMillan case to 2D, round colliding beams. **Require non-Laplacian potentials to realize**
  - Round colliding beams possess 1 invariant – VEPP-2000 at BINP (Novosibirsk, Russia) commissioned in 2006. Record-high beam-beam tune shift ~0.25 attained in 2013
- Danilov, Shiltsev (1998) – Non-linear low energy electron lenses suggested, FNAL-FN-0671
- Chow, Cary (1994)
- ✓ Nonlinear Integrable Optics: Danilov and Nagaitsev solution for nonlinear lattice with 2 invariants of motion that **can be implemented with Laplacian potential**, i.e. with special magnets – *Phys. Rev. ST Accel. Beams 13, 084002 (2010)*

What is Common to These Solutions?

- One begins from **a conventional linear lattice accelerator** (albeit specially designed and carefully controlled) and then **adds the special nonlinear element** (Laplacian or E-Lens) and to make the lattice 2-D integrable.
  - **Does not require much new technology – a significant portion of accelerator circumference is made with common quadrupoles and dipoles**

2D Generalization of McMillan Mapping



SOME THOUGHTS ON STABILITY
IN NONLINEAR PERIODIC FOCUSING SYSTEMS

Edwin M. McMillan

September 5, 1967

- 1D – thin lens kick $f(x) = -\dfrac{Bx^2 + Dx}{Ax^2 + Bx + C}$

$$x_i = p_{i-1}$$
$$p_i = -x_{i-1} + f(x_i)$$

$$Ax^2p^2 + B\left(x^2p + xp^2\right) + C\left(x^2 + p^2\right) + Dxp = \text{const}$$

- 2D – a thin lens solution can be carried over to 2D case in axially symmetric system

1. The ring with transfer matrix

$$\begin{pmatrix} cI & sI \\ -sI & cI \end{pmatrix} \begin{pmatrix} 0 & \beta & 0 & 0 \\ -\frac{1}{\beta} & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta \\ 0 & 0 & -\frac{1}{\beta} & 0 \end{pmatrix}$$

$$c = \cos(\phi)$$
$$s = \sin(\phi)$$
$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

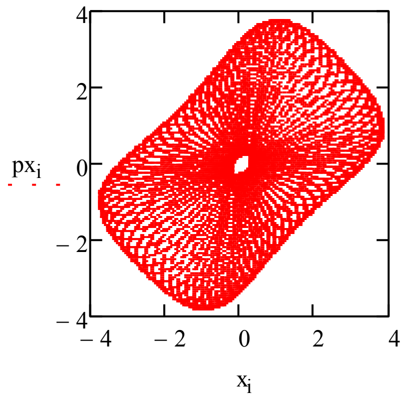
2. Axially-symmetric thin kick

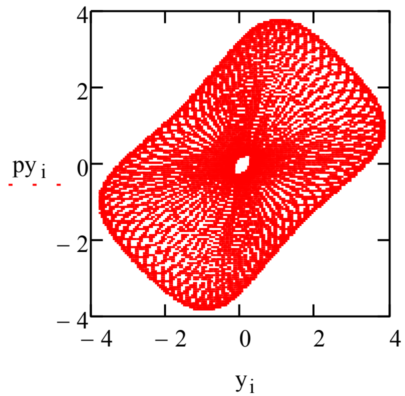
$$\theta(r) = \frac{kr}{ar^2 + 1}$$

can be created with electron lens

2D Generalization of McMillan Mapping

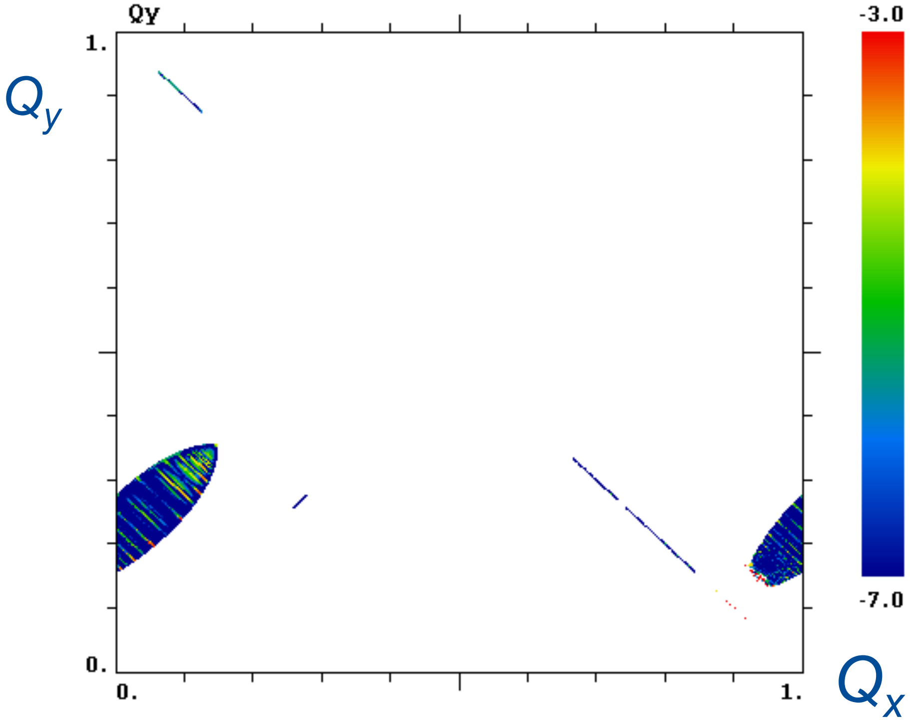
- The system possesses two integrals of the motion (transverse):
  - Angular momentum: $xp_y - yp_x = const$
  - McMillan-type integral, quadratic in momentum

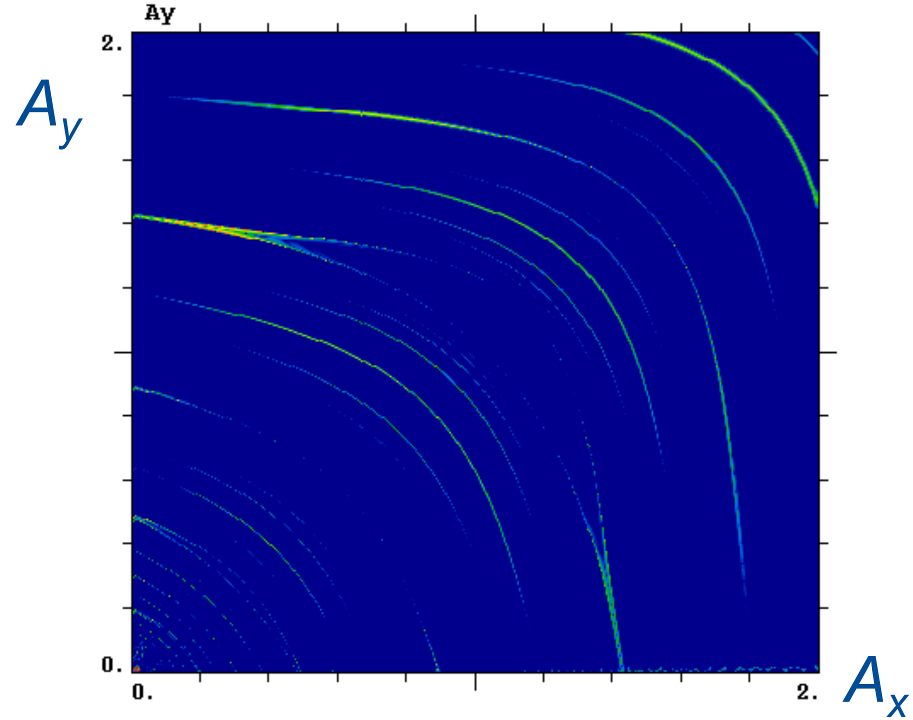




- For large amplitudes, the fractional tune is 0.25
- For small amplitude, the electron (defocusing) lens can give a **tune shift of –0.25 per cell !**

2D Generalization of McMillan Mapping



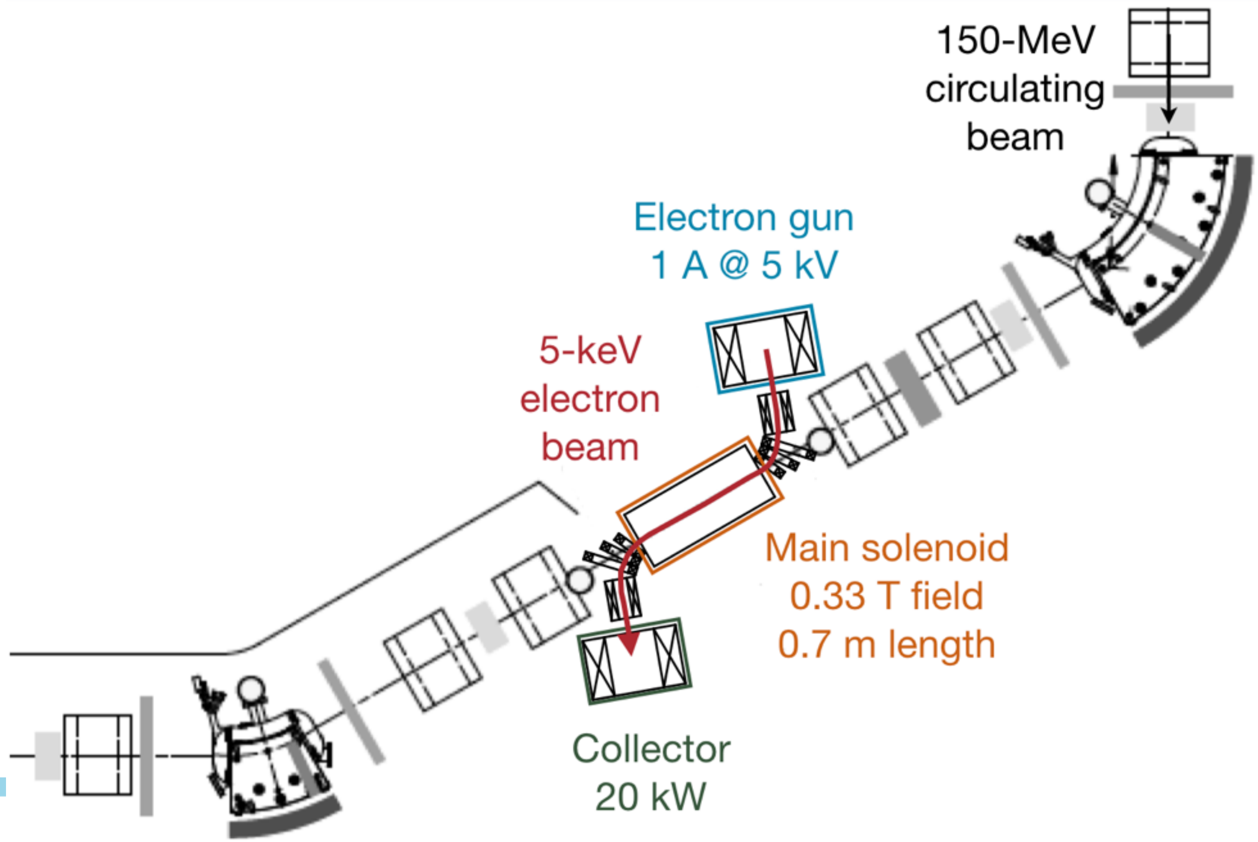


Numerical tracking + Frequency Map Analysis

- All excited resonances have the form k · (Q_x + Q_y) = m. They do not cross each other, so there are no stochastic layers and diffusion

Implementation with Electron Lens

- Low-energy electron beam with nonlinear density, confined in strong longitudinal magnetic field interacts with circulating beam
- Used at Tevatron and RHIC



Another Class of Solutions

1. Remove time dependence from Hamiltonian thus making it an integral of the motion
   – One integral already gives a better degree of regularity in the motion (round colliding beams, 1/2-integer working point in colliders, crab-crossing at DAΦNE)
2. Shape the nonlinear potential to find a second integral

Time-Independent Hamiltonian

- Start with a Hamiltonian

$$H = \frac{p_x^2}{2} + \frac{p_y^2}{2} + K(s)\left(\frac{x^2}{2} + \frac{y^2}{2}\right) + V(x, y, s)$$

- Choose s-dependence of the nonlinear potential such that *H* is time-independent in normalized variables

$$z_N = \frac{z}{\sqrt{\beta(s)}},$$

$$p_N = p\sqrt{\beta(s)} - \frac{\beta'(s)z}{2\sqrt{\beta(s)}},$$

$$H_N = \frac{p_{xN}^2 + p_{yN}^2}{2} + \frac{x_N^2 + y_N^2}{2} + \beta(\psi)V\left(x_N\sqrt{\beta(\psi)}, y_N\sqrt{\beta(\psi)}, s(\psi)\right)$$

$$H_N = \frac{p_{xN}^2 + p_{yN}^2}{2} + \frac{x_N^2 + y_N^2}{2} + U(x_N, y_N, \psi)$$

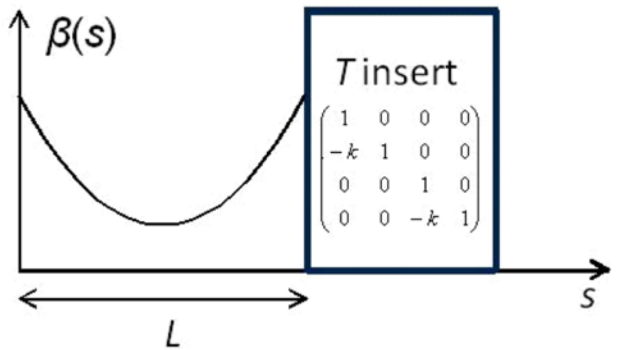
- This results in *H* being the integral of motion
- Note there was no requirement on *V* – can be made with any conventional magnets, i.e. octupoles

Recipe for Time-Independent Hamiltonian

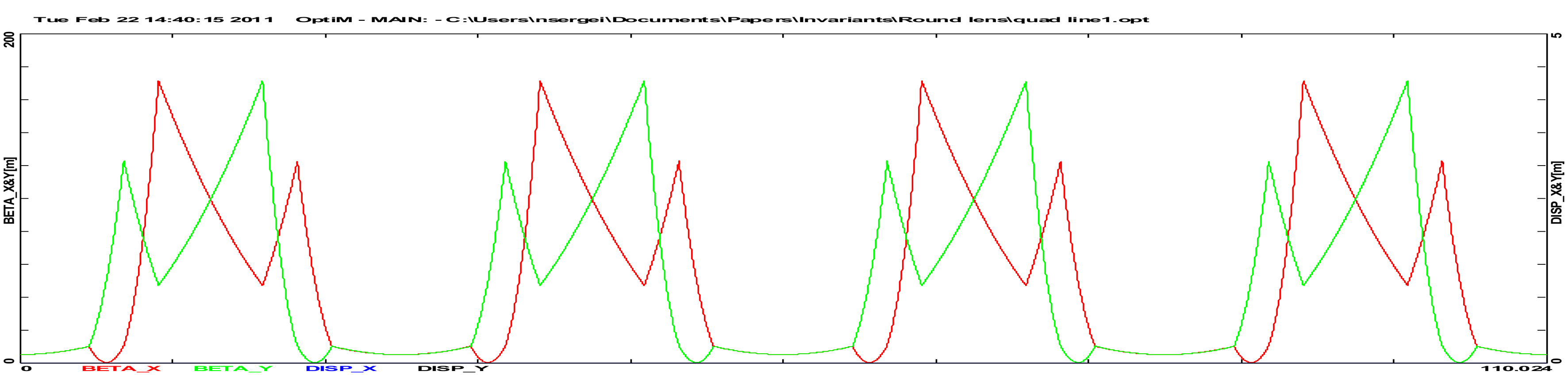
1 Start with a round axially-symmetric *linear* lattice (FOFO) with the element of periodicity consisting of

   a. Drift L

   b. Axially-symmetric focusing block "T-insert" with phase advance n×π



2 Add special nonlinear potential *V(x,y,s)* in the drift



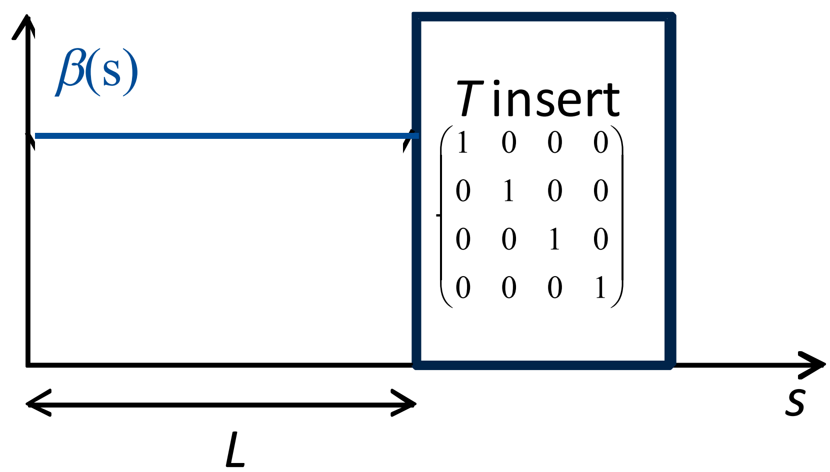
Axially Symmetric Time-Independent Hamiltonian

- Linear optical system consisting of
  a. Section with a constant beta function.
     - For a 150-MeV electron beam, a solenoid 0.5T gives 2-m beta functions (constant over the solenoid length, L)

  b. Matched focusing block "T-insert" with a phase advance of n × π (+ arbitrary rotation)
  – The fractional tunes are: $Q_0 = \frac{L}{2\pi\beta}$ or $0.5 + \frac{L}{2\pi\beta}$

- Add axially-symmetric nonlinear potential in the drift (electron lens)

$$V(x,y,s) = V(x,y)$$



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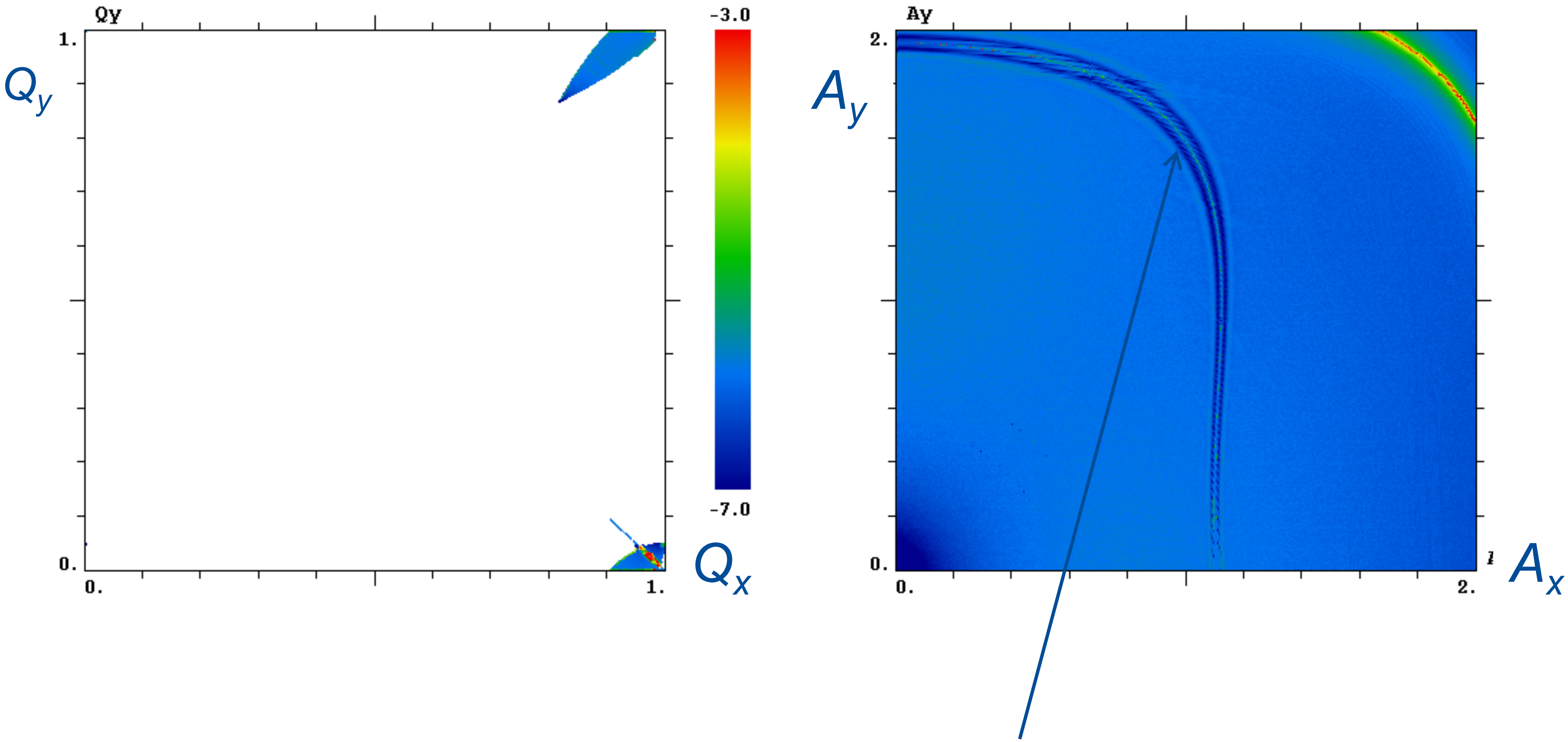
Axially Symmetric Time-Independent Hamiltonian

- The system is integrable. Two integrals of motion (transverse):
  – Angular momentum: $xp_y - yp_x = const$
  – The total transverse energy (Hamiltonian is time-independent).
- A very interesting case is near (just above) the integer resonance
  – The system can lose linear (small amplitude) stability but retain the large amplitude stability

Axially Symmetric Time-Independent Hamiltonian



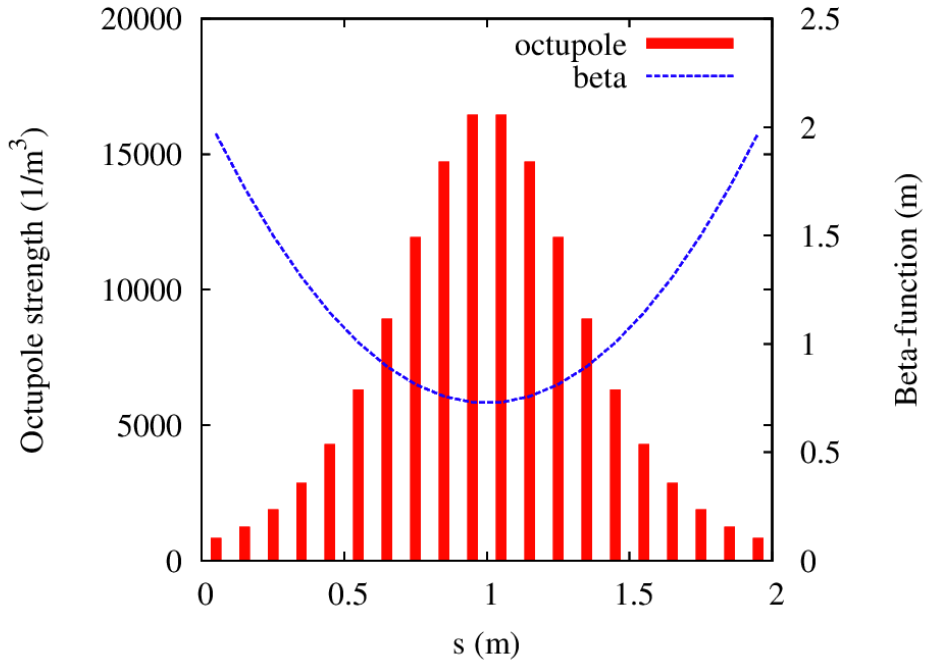
The footprint with one defocusing lens crosses integer resonance

Henon-Heiles Type Systems

- For example, build *V* with Octupoles

$$V(x,y,s)=\frac{\kappa}{\beta(s)^3}\left(\frac{x^4}{4}+\frac{y^4}{4}-\frac{3x^2y^2}{2}\right)$$

$$U=\kappa\left(\frac{x_N^4}{4}+\frac{y_N^4}{4}-\frac{3y_N^2x_N^2}{2}\right)$$



$$H=\frac{1}{2}(p_x^2+p_y^2)+\frac{1}{2}(x^2+y^2)+\frac{k}{4}\left(x^4+y^4-6x^2y^2\right)$$

- Only one integral of motion – *H*
- Tune spread limited to ~12% of Q_0

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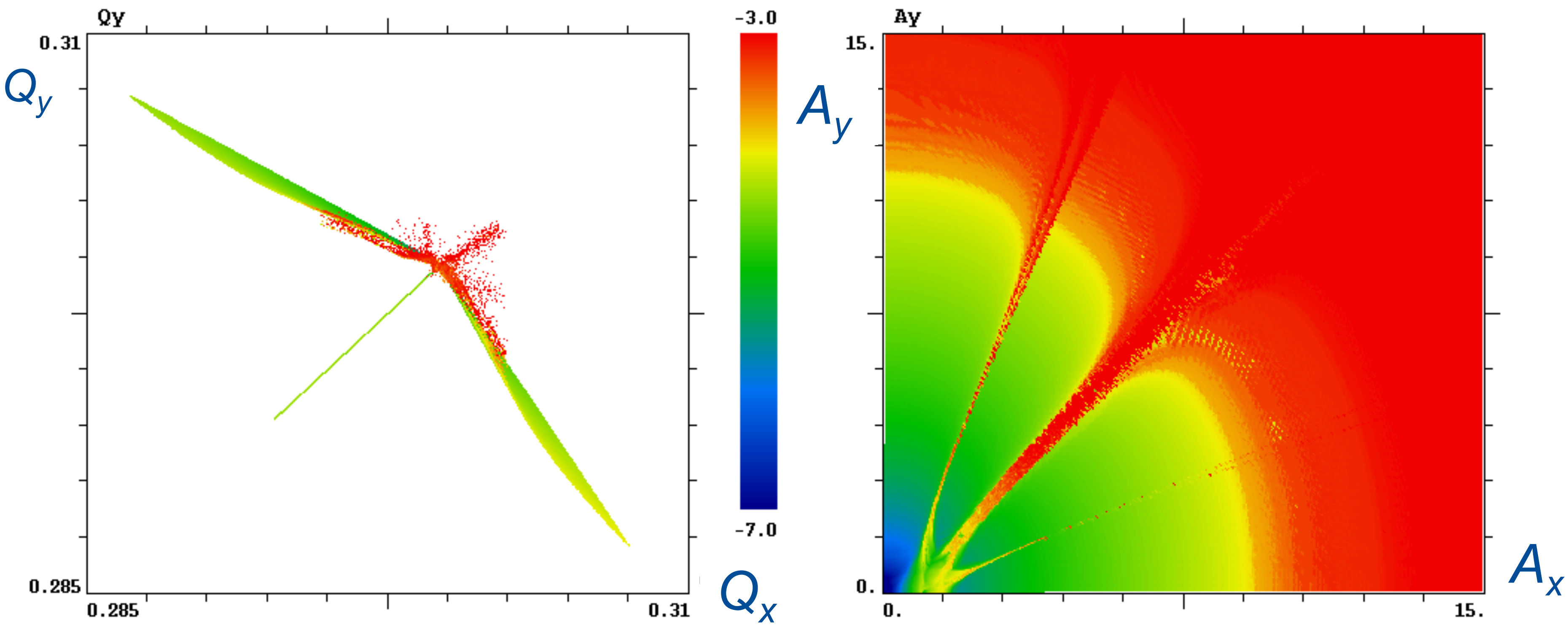
Henon-Heiles Type System with Octupoles



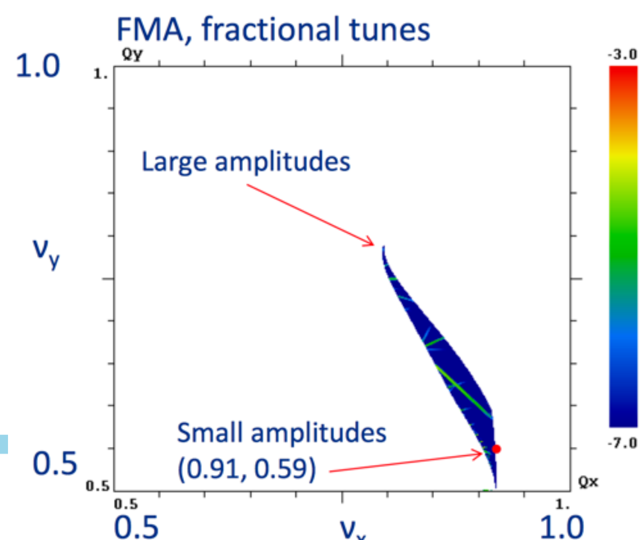
- While dynamic aperture is limited, the attainable tune spread is large ~0.03 – compare to 0.001 created by LHC octupoles

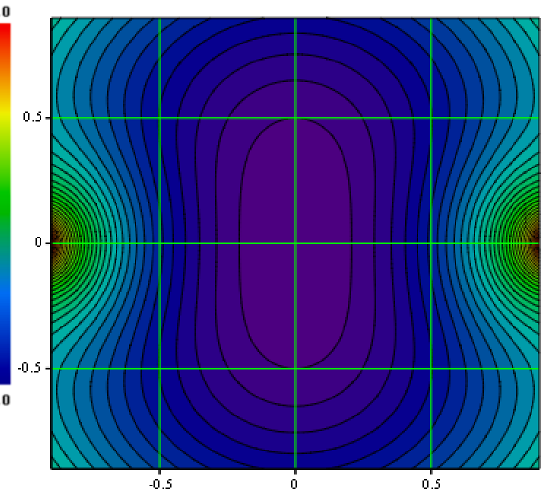
Special Potential – Second Integral of Motion

- Find potentials that result in the Hamiltonian having a second integral of motion quadratic in momentum
  - All such potentials are separable in some variables (cartesian, polar, elliptic, parabolic)
  - First comprehensive study by Gaston Darboux (1901)
- Darboux equation

$$xy\left(U_{xx}-U_{yy}\right)+\left(y^2-x^2+c^2\right)U_{xy}+3yU_x-3xU_y=0$$

  - General solution was found, which satisfies the Laplace equation (*Phys. Rev. ST Accel. Beams 13, 084002, 2010*)





Stability of Nonlinear Lattices

Nonlinear systems can be more stable!

- 1D systems: non-linear (unharmonic) oscillations can remain stable under the influence of periodic external force perturbation. Example: $\ddot{z} + \omega_0^2 \sin(z) = a \sin(\omega_0 t)$
- 2D: The resonant conditions $k\omega_1(J_1, J_2) + l\omega_2(J_1, J_2) = m$ are valid only for certain amplitudes.
- Nekhoroshev's condition guaranties detuning from resonance and, thus, stability.
  - *An Exponential Estimate of the Time of Stability of Nearly-Integrable Hamiltonian Systems*. Russian Math. Surveys 32:6 (1977) from Uspekhi Mat. Nauk 32:6 (1977)

Stability of Nonlinear Lattices

- Proposed solutions have very tight tolerances on the implementation
  - Linear maps (1% in β, 0.001 in phase)
  - Nonlinear potential (3D implementation)
- We perform rigorous studies of stability to
  - Chromatic aberrations
  - Nonlinearities and other imperfections in the arcs
  - Effect of 3^{rd} degree of freedom
  - Space charge
- Some implementations are more stable
  - Elliptic potential does not satisfy Nekhoroshev's criterion
  - Hamiltonian for Electron lens is steep

Investigation of kinematic nonlinearities and 2nd/3rd-order dipole effects in the IOTA ring (nonlinear insert *ON*)

<u>IOTA Lattice (2.5 MeV p)</u>

2 nonlinear inserts *ON*

L=1.8 m, t=0.45,

c = 0.009 $m^{1/2}$, μ_0=0.3

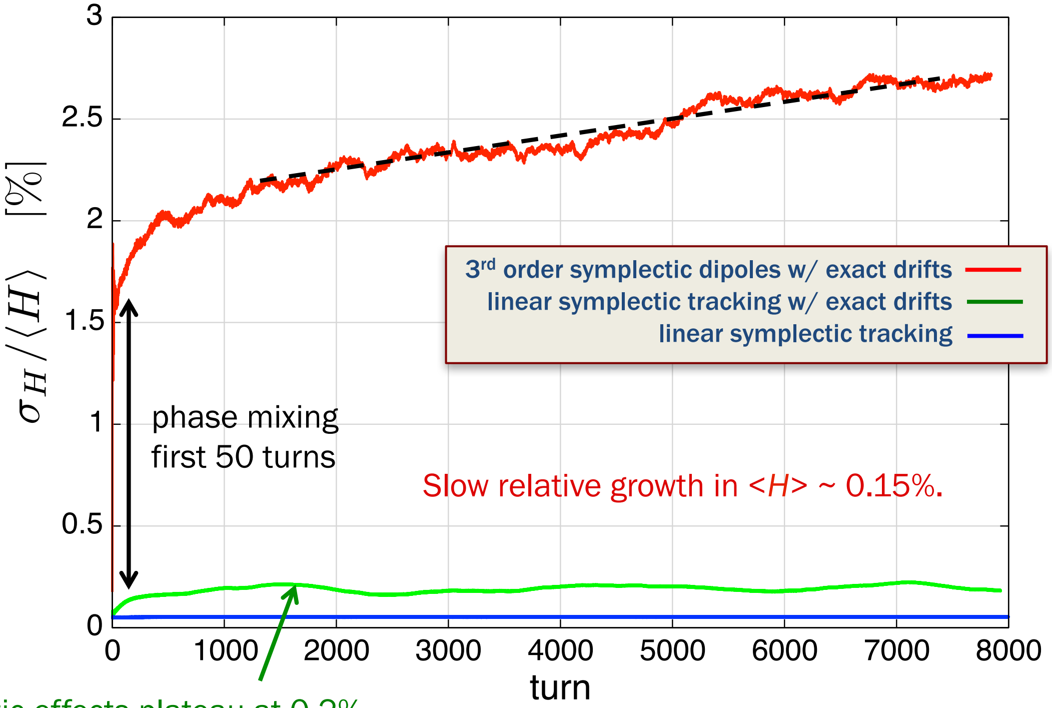
Matched nonlinear KV beam

ε_0 = 3.9 mm-mrad, σ_δ = 0

Rapid mixing over the first 2-3 turns, followed by slow diffusion.

- constant if *H* invariant is preserved
- zero for a "nonlinear KV" distribution

Diffusion of the H invariant (IMPACT-Z)



The corresponding value of $\sigma_I/\langle I\rangle$ (for the second invariant) grows by 1% over 8000 turns.

Expected to be less problematic for a large ring such as the proposed Rapid Cycling Synchrotron.

Proof of Principle Experiments – IOTA

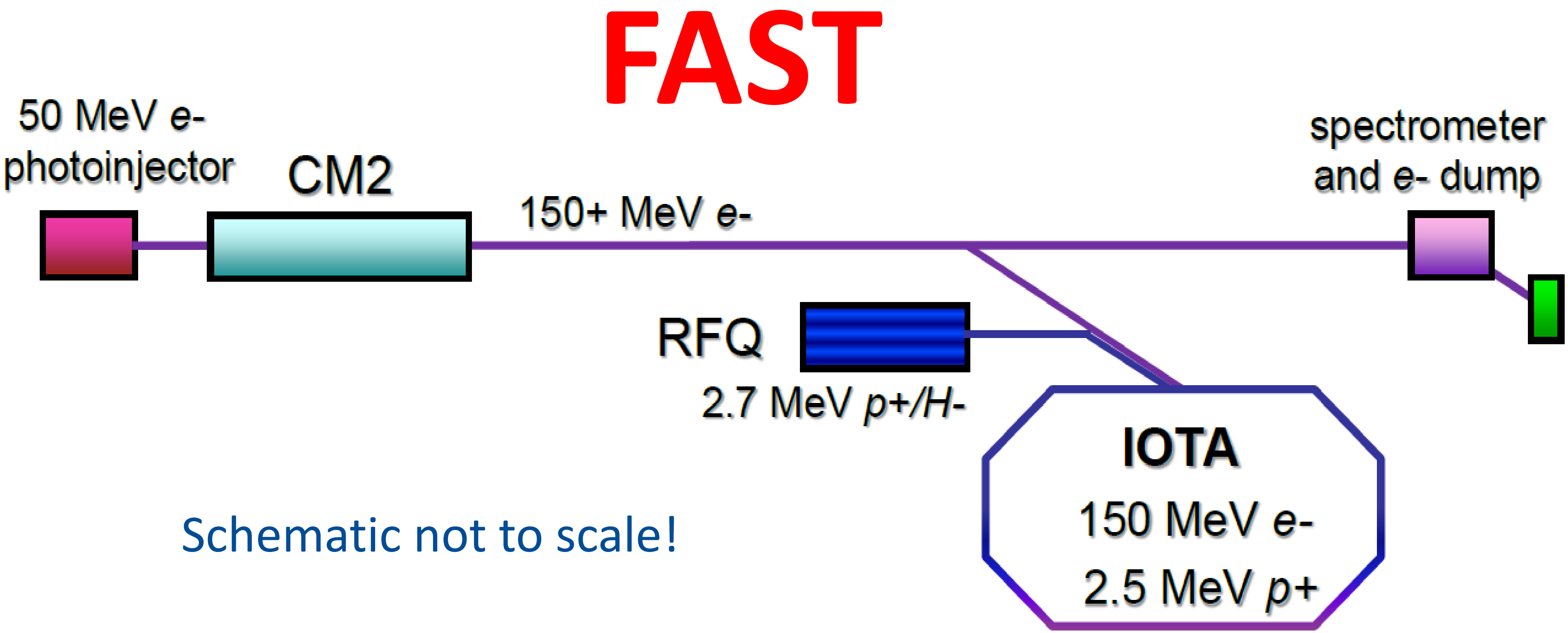
- We are building the **Integrable Optics Test Accelerator**
  - To test the Nonlinear Integrable Optics
  - To become a machine for proof-of-principle R&D

IOTA Physics Drivers

1. **Nonlinear Integrable Optics –** Experimental demonstration of NIO lattice in a practical accelerator
2. **Space Charge Compensation –** Suppression of SC-related effects in high intensity circular accelerators
   – Nonlinear Integrable Optics
   – Electron lenses
   – Electron columns
   – Circular betatron modes
3. **Optical Stochastic Cooling** – Proof-of-principle demonstration
4. **Beam collimation –** Technology development for hollow electron beam collimation
5. **Electron Cooling** – Advanced techniques

- **Laser-Plasma Accelerator –** Demonstration of injection into synchrotron
- **Quantum Physics –** Localization of single electron wave function

Fermilab Accelerator Science and Technology Facility



Schematic not to scale!

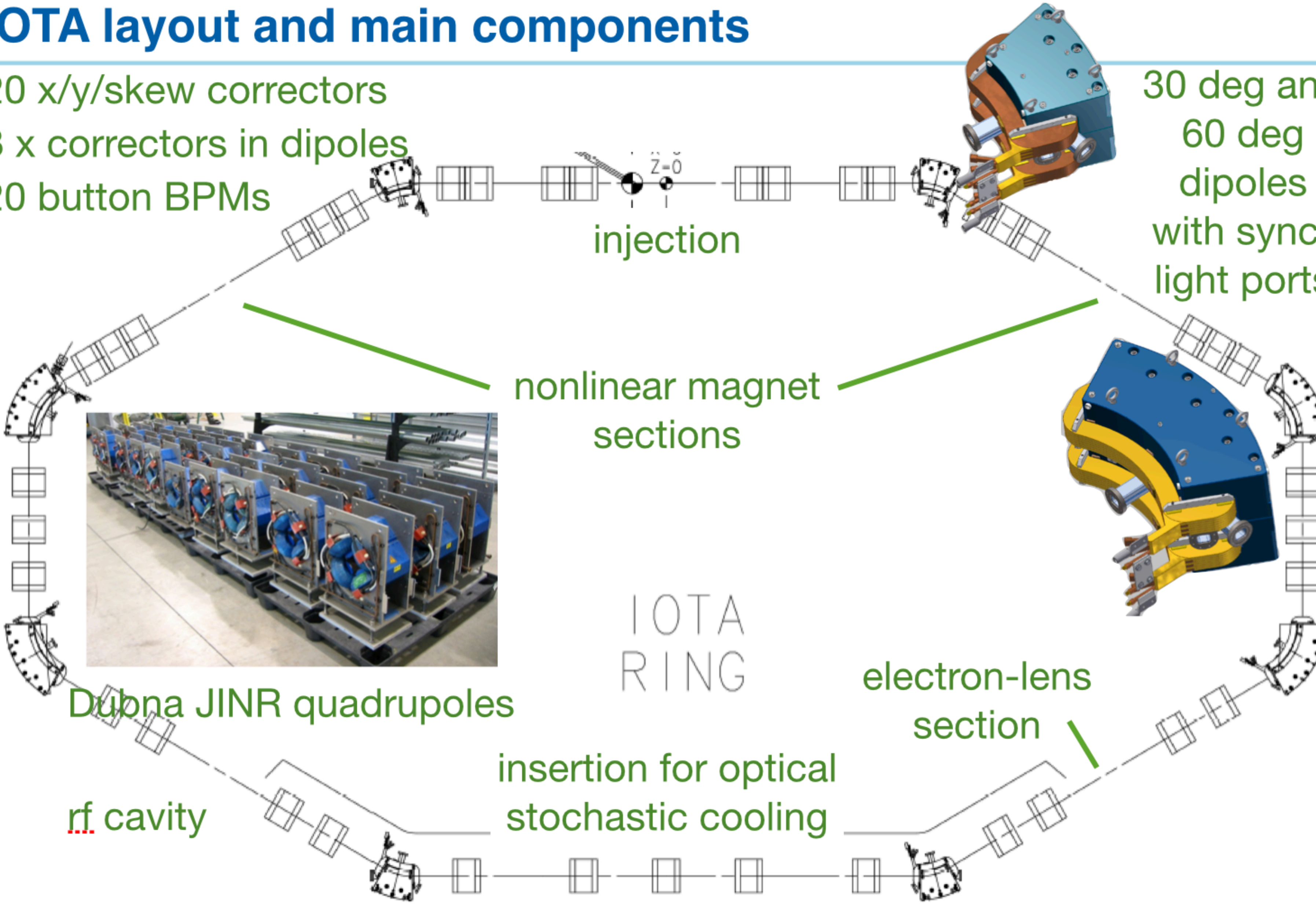
IOTA layout and main components

20 x/y/skew correctors

8 x correctors in dipoles

20 button BPMs

30 deg and 60 deg dipoles with sync-light ports



injection

nonlinear magnet sections

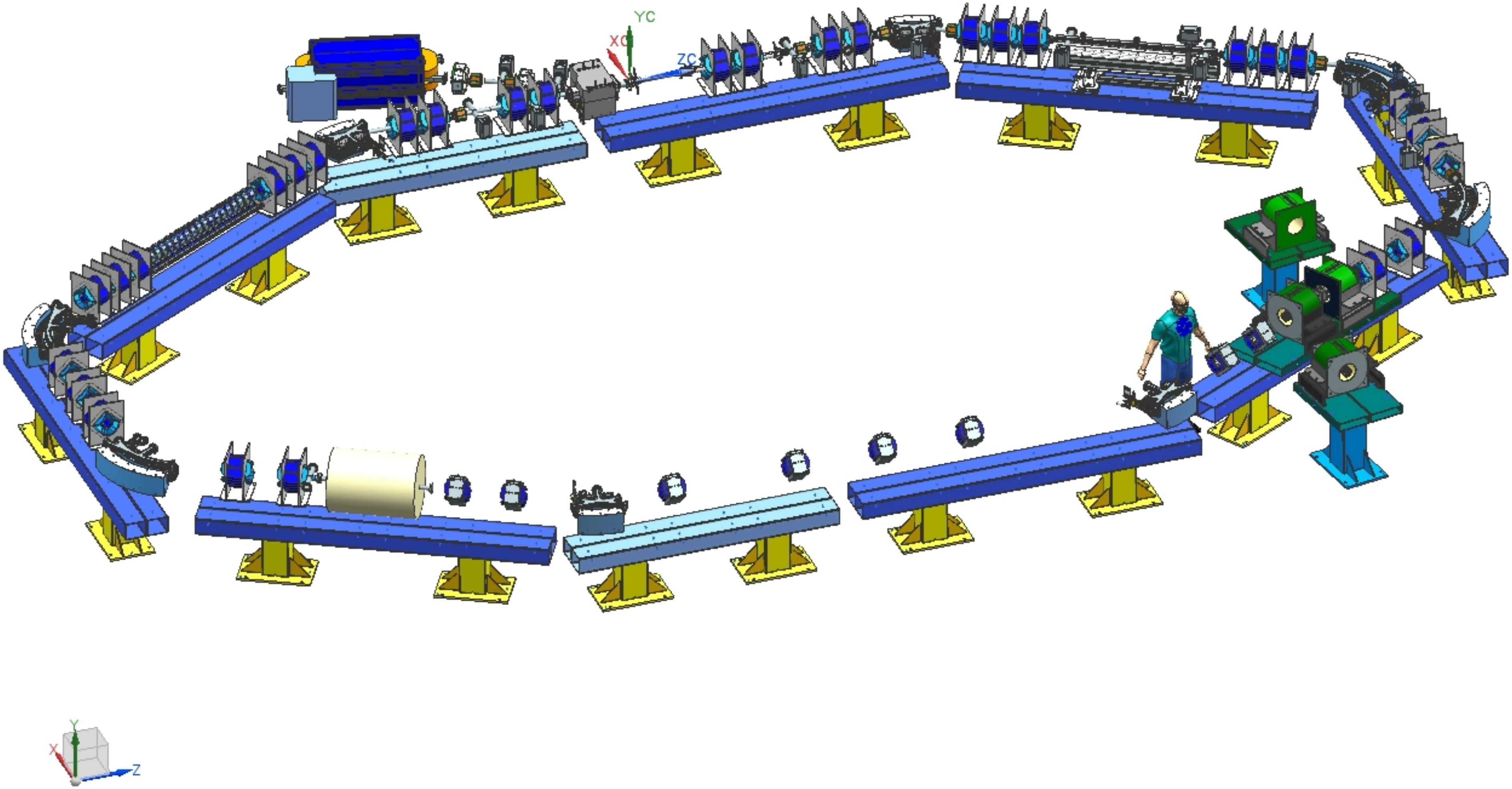
Dubna JINR quadrupoles

electron-lens section

insertion for optical stochastic cooling

rf cavity

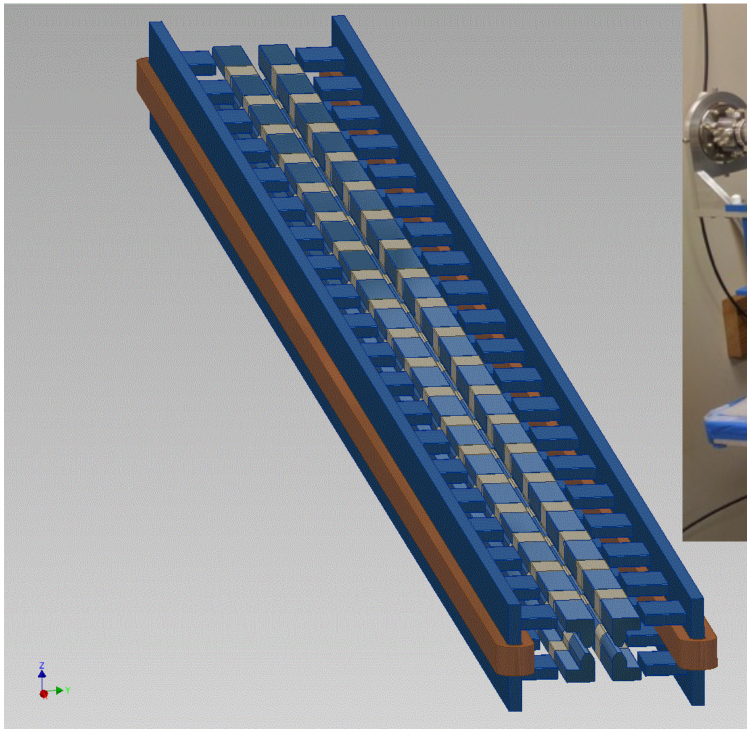
IOTA Layout



IOTA Layout

Nonlinear Magnet

- Joint effort with RadiaBeam Technologies (Phase I and II SBIR)



FNAL Concept: 2-m long nonlinear magnet

RadiaBeam full 1.8-m magnet designed, fabricated and delivered to IOTA in Phase II

IOTA Goals for Integrable Optics

The IOTA experiment has the **goal to demonstrate the possibility to implement nonlinear integrable optics** with a large betatron frequency spread $\Delta Q > 1$ and stable particle motion **in a realistic accelerator design**

IOTA Staging – Phase I

Phase I will concentrate on the academic aspect of single-particle motion stability using e-beams

- **Achieve large nonlinear tune shift/spread** without degradation of dynamic aperture **by "painting"** the accelerator aperture **with a "pencil" beam**
- Suppress strong lattice resonances = cross the integer resonance by part of the beam without intensity loss
- Investigate stability of nonlinear systems to perturbations, develop practical designs of nonlinear magnets
- The measure of success will be the achievement of high nonlinear tune shift = 0.25

IOTA Construction and Research Timeline

	Electron Injector	Proton Injector	IOTA Ring
FY15	20 MeV *e-* commiss'd beam tests	Re-assembly began @MDB	50% IOTA parts ready
FY16	50 MeV *e-* commiss'd beam tests	50 keV *p+* commiss'd	IOTA parts 80+% ready
FY17	150-300 MeV *e-* beam commissioning/tests *	2.5 MeV *p+* commiss'd beam tests @ MDB	IOTA fully installed first beam ? *
FY18	*e-* injector for IOTA + other research	*p+* RFQ moved from MDB to FAST *	IOTA commiss'd with *e-* **Research starts (NL IO)**
FY19	*e-* injector for IOTA + other research	2.5 MeV *p+* commiss'd beam tests	**IOTA research with *e-*** IOTA commiss'd with *p+*
FY20	*e-* injector for IOTA + other research	*p+* injector for IOTA	**IOTA research with *p+****

beam operations

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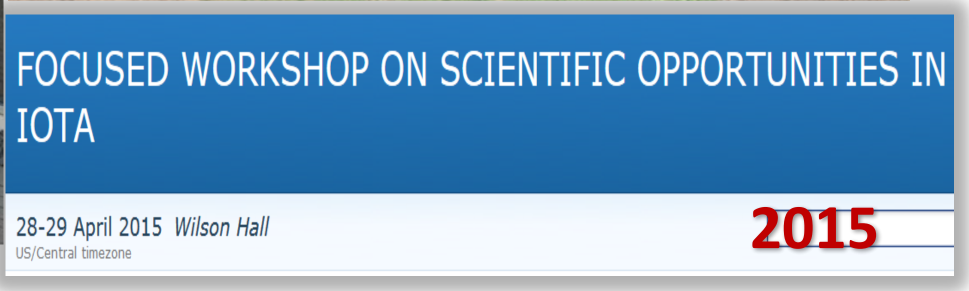
Collaboration Today

- **25 Partners:**
  - ANL, Berkeley, BNL, BINP, CERN, Chicago, Colorado State, IAP Frankfurt, JINR, Kansas, LANL, LBNL, ORNL, Maryland, Universidad de Guantajuato Mexico, Michigan State, NIU, Oxford, RadiaBeam Technologies, RadiaSoft LLC, Tech-X, Tennessee, Vanderbilt
- **NIU-FNAL:** Joint R&D Cluster



Training and University Collaboration

- Excellent connection to the university community through the Joint Fermilab/University PhD program
  - Already 9 graduate students doing thesis research at FAST/IOTA
- Opportunities for summer internships

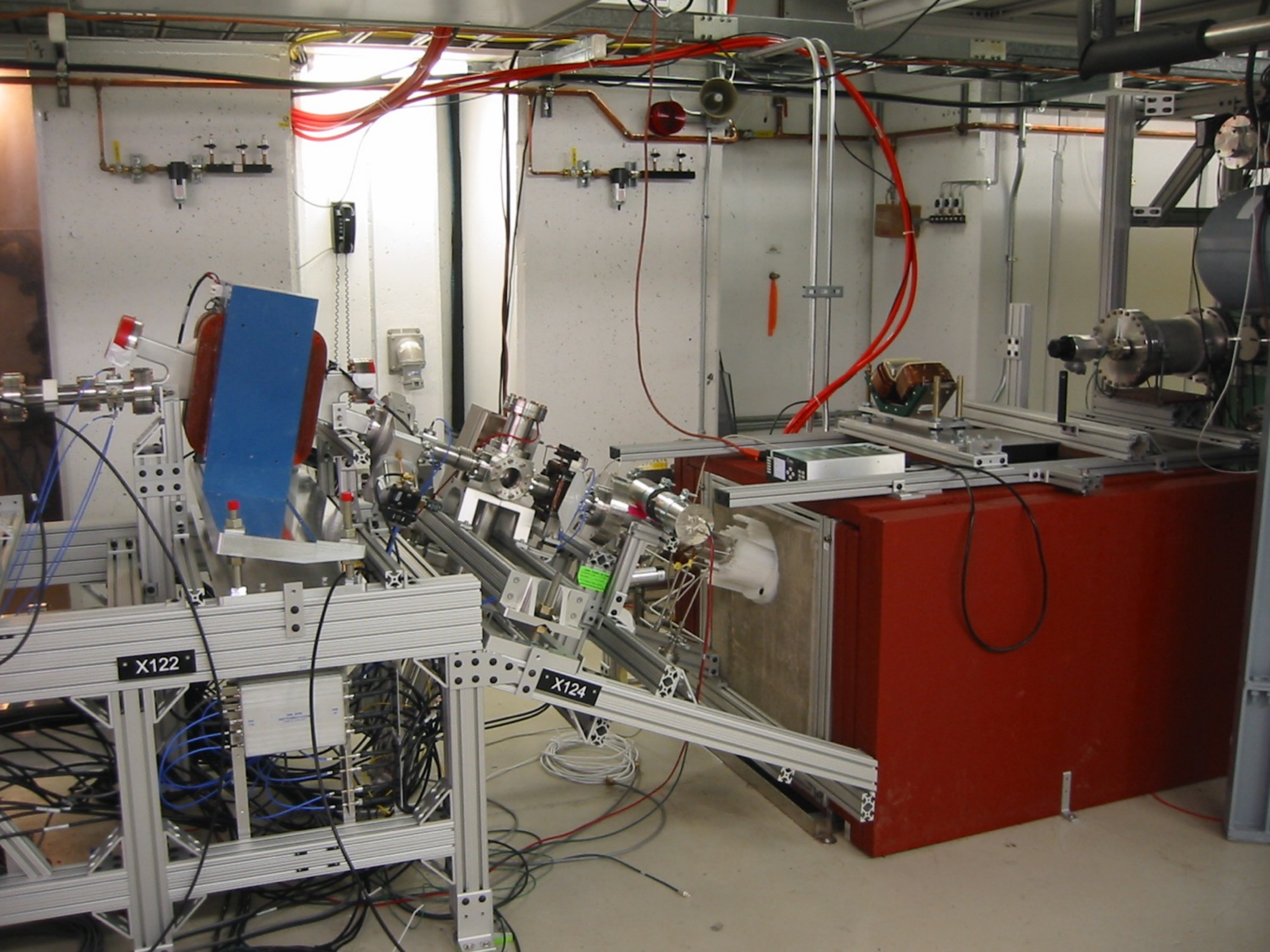
Summary

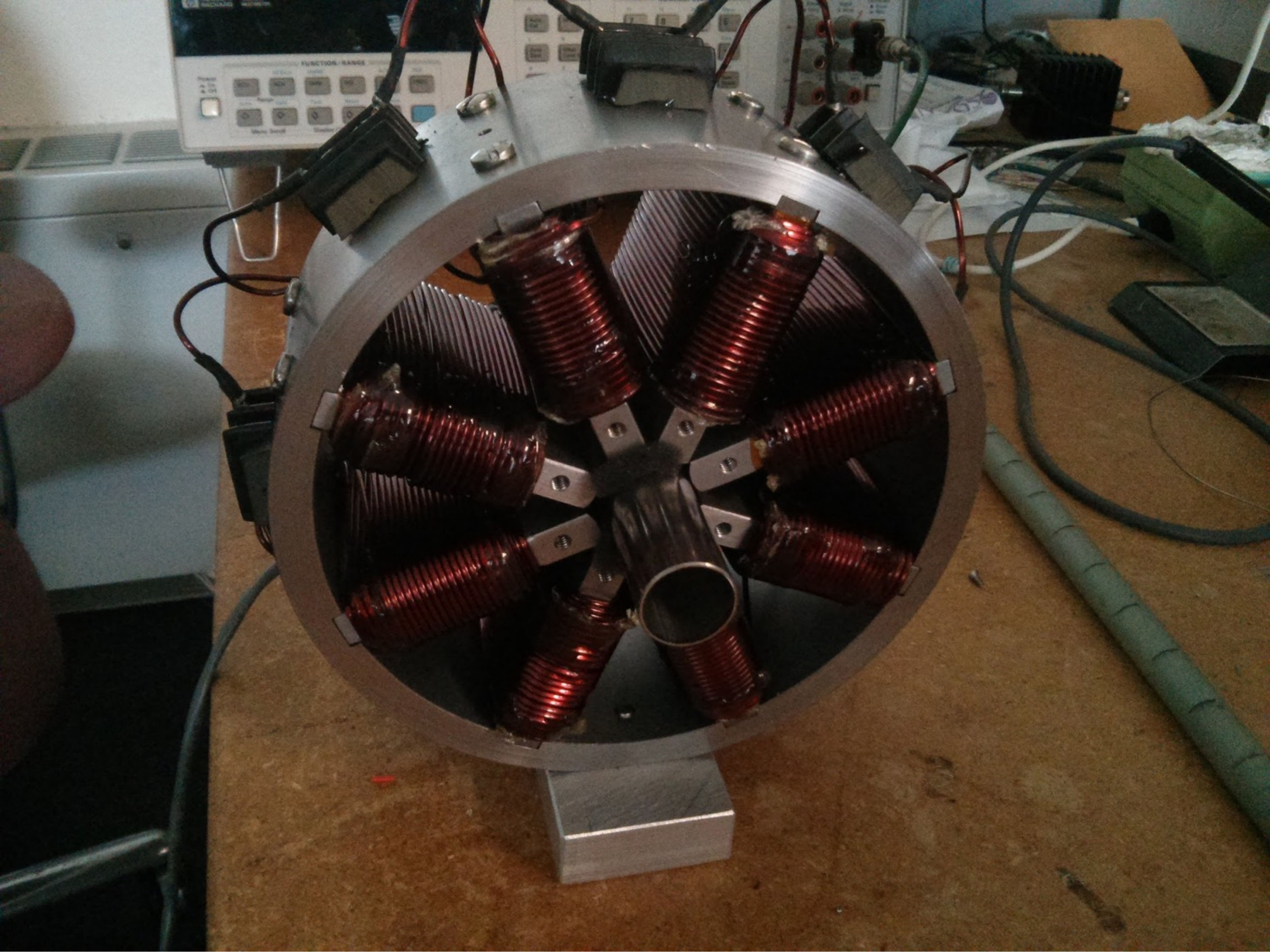
- IOTA/FAST offers a **vast and unique program in accelerator science**
- IOTA/FAST experiments are a great opportunity to explore something **truly novel with circular accelerators**
- IOTA/FAST will be a strong **driver of national and international collaboration and training**

IOTA Parameters

Nominal kinetic energy	e^-: 150 MeV, p+: 2.5 MeV
Nominal intensity	e^-: 1×10^9, p+: 1×10^{11}
Circumference	40 m
Bending dipole field	0.7 T
Beam pipe aperture	50 mm dia.
Maximum b-function (x,y)	12, 5 m
Momentum compaction	0.02 ÷ 0.1
Betatron tune (integer)	3 ÷ 5
Natural chromaticity	-5 ÷ -10
Transverse emittance r.m.s.	e^-: 0.04 μm, p+: 2μm
SR damping time	0.6s (5×10^6 turns)
RF V,f,q	e^-: 1 kV, 30 MHz, 4
Synchrotron tune	e^-: 0.002 ÷ 0.005
Bunch length, momentum spread	e^-: 12 cm, 1.4×10^{-4}


EXIT
DANGER
X122
X124


X122
X124


FUNCTION / RANGE